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NUMERICAL SIMULATION OF JAMMING TRANSITION IN GRANULAR SYSTEM UNDER CYCLIC COMPRESSION USING SMOOTH PARTICLE HYDRODYNAMICS

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ABSTRACT

The jamming of granular materials, which indicates how disordered particle systems change from mechanically unstable to stable states, has attracted significant recent interest due, but not limited, to the appearance of jamming transition or similar behavior in a broad variety of systems. Recent experiments on jamming transition have revealed the relationship between mean coordination number and packing fraction for different jammed states. In this paper the jamming states of two dimensional granular materials under cyclic compression using Smooth Particle Hydrodynamics (SPH) approach is numerically investigated. The SPH method allows one to study the stress developed within individual granular particles of arbitrary shape. In this study the granular system is cyclically and isotropically compressed or expanded. The system undergoes a range of jamming states over a large number of cycles. We measure the evolution of global pressure, mean coordination number, and packing fraction. The force chains and probability density function of force for different compression cycles are also investigated.

Keywords: Smooth Particle Hydrodynamics; Jamming; Granular Material; Contact Model, Force Chain

INTRODUCTION

Granular materials, such as sand, are collection of discrete macroscopic particles which exhibit a wide range of interesting macroscopic behavior such as pile formation, fluid-flow like behavior and fracture. They are important in many industrial applications such as mining, construction, agriculture and packing. They also play an important role in geological processes such as landslide, avalanche, erosion, sedimentation and plate tectonics. One particular aspect of granular material is the phenomenon of jamming where randomly organized system of particles changes from mechanically unstable states to stable states. Jamming phenomena is also observed in colloids, foams and glass transition in molecular liquids. In most of these cases the system starts from an unjammed state and gradually transition to jammed state. Sometimes the system undergoes several transitions between jammed and unjammed states. In this paper the jammed states of two dimensional granular materials is numerically investigated by subjecting the system to consecutive compression cycles.

There are a number of simulation methods developed so far in order to capture the behavior of granular materials. These methods can be divided into two categories – discrete and continuum. Discrete method is by far the most widely used method for simulating granular media. In discrete methods, such as DEM (Discrete Element Method), behavior of each particle that makes up the granular system is captured by directly simulating the interaction between them. However, DEM cannot provide enough resolution to study the stresses developed in individual particles. In this paper, Smoothed Particle Hydrodynamics (SPH) method is used to simulate the interaction of granular particles. In SPH, each granular particle is made up of several SPH particles which can accurately account for the stress generated within each granular particle. In addition contact forces are calculated using an advanced formulation that accounts for the contact geometry and force distribution around the contact area.

SPH method, introduced by Lucy [1], Gingold [2, 3] and Monaghan [4, 5], was originally developed for solving astrophysics problem. This method has the ability to handle large deformation especially when comes to solid bodies. Hence it has been applied to hyper-velocity impact [5], explosion [6] and fluid

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dynamics [7]. When SPH is applied to high velocity impact problems boundary and contact properties on the contact surface are ignored because of their insignificant contribution. However, for low velocity impact or small deformation, as mentioned in [8], neglecting contact conditions can arise several problems. Some of them include ghost stress, which arises due to incorrect inclusion of boundary particles within a smoothing length, false tensile force during separation and normalization of kernel not satisfied. In order to avoid such problems, [8] developed a method that addresses problems involving small deformation as well as large deformation for low velocity impact. In this paper their method is adopted for simulation of granular particles.

NOMENCLATURE

A	area of contact, m^2
c	sound speed, m/s
C	sound speed, m/s
d_0	diameter of particle, m
e	specific internal energy, J/kg
E	Elastic modulus, Pa
F	force on particle, N
G	Shear modulus, Pa
h	smoothing length, m
m	mass of particle, kg
p	hydrostatic pressure, N/m^2
r	distance vector, m
R	normalized distance
s	deviatoric stress tensor, N/m^2
v	velocity, m/s
V	volume of particle, m^3
x	distance vector, m
W	kernel function

Greek Symbols

α_d	normalization factor
β	constant
Γ	EOS parameter
ε	tensile instability coefficient
η	EOS parameter
θ	angle between normal vectors
λ	penalty parameter
Π	artificial viscosity term
ρ	density, kg/m^3
$\boldsymbol{\sigma}$	stress tensor, N/m^2
ψ	color function

METHODS AND PHYSICAL MODELS

THE EQUATIONS OF MOTION

The mass, momentum and energy conservation equations from continuum mechanics are given as,

$$\frac{D\rho}{Dt} = -\rho \frac{\partial v^\alpha}{\partial x^\alpha} \quad (1)$$

$$\frac{Dv^\alpha}{Dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta} \quad (2)$$

$$\frac{De}{Dt} = \frac{\sigma^{\alpha\beta}}{\rho}\frac{\partial v^\alpha}{\partial x^\beta} \quad (3)$$

where ρ the density, v^α the velocity component, $\sigma^{\alpha\beta}$ the total stress tensor and e is the specific internal energy.

The total stress tensor $\sigma^{\alpha\beta}$ is made up of two parts, the hydrostatic pressure p and the deviatoric shear stress s.

$$\sigma^{\alpha\beta} = -p\delta^{\alpha\beta} + s^{\alpha\beta} \quad (4)$$

The stress rate obtained from the constitutive relation must be invariant with respect to rigid body rotation when large deformation is involved. In this paper, the Jaumann stress rate is adopted for this purpose as,

$$\dot{s}^{\alpha\beta} - s^{\alpha\gamma}\dot{R}^{\beta\gamma} - s^{\gamma\beta}\dot{R}^{\alpha\gamma} = 2G\dot{e}^{\alpha\beta} \quad (5)$$

where $\dot{R}^{\alpha\beta}$ is the rotation rate tensor defined as

$$\dot{R}^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial v^\alpha}{\partial x^\beta} - \frac{\partial v^\beta}{\partial x^\alpha}\right) \quad (6)$$

SPH FORMULATION

The SPH or Smoothed Particle Hydrodynamics is a meshless method where the governing equations are solved by discretizing the entire system using finite number of particles that carry individual mass and occupy certain space. These particles are the mathematical interpolation point themselves. The material properties of the particles are calculated from their relationship with neighboring particles using the kernel function for interpolation.

In this paper the elastic SPH model formulated by [9] are used. The SPH approximation for mass, momentum and energy equation take the following form,

$$\frac{D\rho_i}{Dt} = \sum_{j=1}^{N} m_j \left(v_i^a - v_j^a\right)\frac{\partial W_{ij}}{\partial x_i^\alpha} \quad (7)$$

$$\frac{Dv_i^\alpha}{Dt} = \sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2}\right)\frac{\partial W_{ij}}{\partial x_i^\beta} \quad (8)$$

$$\frac{De_i}{Dt} = -\frac{1}{2}\sum_{j=1}^{N} m_j \left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2} + \frac{\sigma_j^{\alpha\beta}}{\rho_j^2}\right)\left(v_i^a - v_j^a\right)\frac{\partial W_{ij}}{\partial x_i^\beta} \quad (9)$$

where m is the mass of individual particle and W_{ij} is the kernel function.

Several different kernel functions have been used in SPH literature. The one used in this paper is the most popular one, the cubic spline function proposed by [4], which has the following form:

$$W(R,h) = \alpha_d \times \begin{cases} \frac{2}{3} - R^2 + \frac{1}{2}R^3 & 0 \le R < 1 \\ \frac{1}{6}(2-R)^3 & 1 \le R < 2 \\ 0 & R > 2 \end{cases} \quad (10)$$

where α_d is the normalization factor which is $15/7\pi h^2$ in two-dimension, h is the smoothing length and R is the distance between particles i and j normalized as $R = r/h$.

ARTIFICIAL VISCOSITY

In SPH literature a dissipative term, $\prod_{ij}$, which is also called artificial viscosity, is introduced into the governing equation to prevent large unphysical oscillation in the numerical solution and to improve numerical stability. The artificial viscosity is incorporated into the momentum equation in the following way:

$$\frac{Dv_i^{\alpha}}{Dt}=\sum_{j=1}^{N}m_j\left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2}+\frac{\sigma_j^{\alpha\beta}}{\rho_j^2}-\prod_{ij}\delta^{\alpha\beta}\right)\frac{\partial W_{ij}}{\partial x_i^{\beta}} \tag{11}$$

The most widely applied artificial viscosity derived by Monaghan [10] is used in this paper.

$$\prod_{ij}=\begin{cases}\dfrac{-\alpha\bar{c}_{ij}\mu_{ij}+\beta\mu_{ij}^2}{\bar{\rho}_{ij}} & v_{ij}\cdot x_{ij}<0\\ 0 & v_{ij}\cdot x_{ij}\geq 0\end{cases} \tag{12}$$

where

$$\mu_{ij}=h\frac{v_{ij}\cdot x_{ij}}{\left|r_{ij}\right|^2+\varepsilon h^2},\quad \bar{c}_{ij}=\frac{1}{2}\left(c_i+c_j\right),\quad \bar{\rho}_{ij}=\frac{1}{2}\left(\rho_i+\rho_j\right) \tag{13}$$

$$x_{ij}=x_i-x_j,\quad v_{ij}=v_i-v_j \tag{14}$$

In the above equation, α and β are constants and taken as unity, c is the sound speed and ε is 0.01.

The energy equation takes the form of

$$\frac{De_i}{Dt}=-\frac{1}{2}\sum_{j=1}^{N}m_j\left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2}+\frac{\sigma_j^{\alpha\beta}}{\rho_j^2}-\prod_{ij}\delta^{\alpha\beta}\right)\left(v_i^a-v_j^a\right)\frac{\partial W_{ij}}{\partial x_i^{\beta}} \tag{15}$$

TENSILE INSTABILITY

In SPH, for solid body deforming under tension numerical instability arises and material can fail unrealistically. This happens because under tension the SPH particles attract each other and tend to clump together. Monaghan [11] and Grey et al. [12] successfully introduced artificial stress method to overcome this problem of tensile instability. Their method involves adding another term to the momentum equation. The additional term comes from the dispersion relations.

$$\frac{dv_i}{dt}=\sum_j m_j\left(\frac{\sigma_i^{\alpha\beta}}{\rho_i^2}+\frac{\sigma_j^{\alpha\beta}}{\rho_j^2}-\Pi_{ij}+\left(R_i^{\alpha\beta}+R_j^{\alpha\beta}\right)f_{ij}^n\right)\frac{\partial W_{ij}}{\partial x_i^{\beta}} \tag{16}$$

where n is the exponent which depends on the smoothing kernel, f_{ij} is the repulsive force term which is given in terms of the kernels by [11] as:

$$f_{ij}=\frac{W_{ij}}{W\left(\Delta p,h\right)} \tag{17}$$

where Δp is the initial particle spacing. In this paper, h is assumed to be constant so that $W\left(\Delta p,h\right)$ is also constant. It is found that best result can be obtained by setting $h=1.5\Delta p$. The [12] suggested a value of 4 for n for best result so that value is used in this work.

For the two-dimensional case used in this paper, the components of the artificial stress tensor $R_i^{\alpha\beta}$ for particle i in the reference coordinate system *(x, y)* are computed from the principal components $R_i'^{xx}$ and $R_i'^{yy}$ by the coordinate transformation:

$$R_i^{xx}=R_i'^{xx}\cos^2\theta_i+R_i'^{yy}\sin^2\theta_i \tag{18}$$

$$R_i^{yy}=R_i'^{xx}\sin^2\theta_i+R_i'^{yy}\cos^2\theta_i \tag{19}$$

$$R_i^{xy}=\left(R_i'^{xx}-R_i'^{yy}\right)\sin\theta_i\cos\theta_i \tag{20}$$

where the angle θ_i is defined as,

$$\tan 2\theta_i=\frac{2\sigma_i^{xy}}{\sigma_i^{xx}-\sigma_i^{yy}} \tag{21}$$

where σ_i^{xx}, σ_i^{yy} and σ_i^{xy} are components of stress tensor of particle i in the reference frame *(x, y)*. The diagonal components of the artificial stress tensor in principle axes are calculated as [12],

$$R_i'^{xx}=\begin{cases}-\dfrac{\varepsilon\sigma_i'^{xx}}{\rho_i^2} & if\,\sigma_i'^{xx}>0\\ 0 & otherwise\end{cases} \tag{22}$$

$$R_i'^{yy}=\begin{cases}-\dfrac{\varepsilon\sigma_i'^{yy}}{\rho_i^2} & if\,\sigma_i'^{yy}>0\\ 0 & otherwise\end{cases} \tag{23}$$

where ε is a constant and is taken as 0.3 which as suggested by [12] is the best value for elastic solid. $\sigma_i'^{xx}$ and $\sigma_i'^{yy}$ are the principal stresses of particle i . They are obtained as,

$$\sigma_i'^{xx}=\sigma_i^{xx}\cos^2\theta_i+2\sin\theta_i\cos\theta_i\sigma_i^{xy}+\sigma_i^{yy}\sin^2\theta_i \tag{24}$$

$$\sigma_i'^{yy}=\sigma_i^{xx}\sin^2\theta_i-2\sin\theta_i\cos\theta_i\sigma_i^{xy}+\sigma_i^{yy}\cos^2\theta_i \tag{25}$$

VELOCITY SMOOTHING

The velocities v of the particles obtained by time integration of the momentum equation are corrected in order to smooth out any unexpected numerical peaks. The correction is done by [13],

$$\hat{v}_i^{\alpha\beta}=v_i^{\alpha\beta}+\tilde{\varepsilon}\sum_{j=1}^{N}\frac{m_j}{0.5\left(\rho_i+\rho_j\right)}\left(v_j^{\alpha\beta}-v_i^{\alpha\beta}\right)W_{ij} \tag{26}$$

The corrected velocities are used to update the position of the particles, while the uncorrected velocities are used for time integration of the momentum equation at the following step.

EQUATION OF STATE

Pressure, density and internal energy of a material are related by the equation of state of the system. In this paper the Mie-Gruneisen equation for solids [9] is used to calculate the pressure arising from the deformation of the material. Table 1 gives the material properties and constants for Mie-Gruneisen EOS of lead.

Table 1 Material Properties

	$\rho_0(g/cm^3)$	c ($\times 10^4$m/s)	S	G(GPa)	E(GPa)	Γ
Lead	1.134	1.19	1.80	5.6	16	2.00

The required equations are

$$p=\left(1-\frac{1}{2}\Gamma\eta\right)p_H(\rho)+\Gamma\rho e \tag{27}$$

where

$$\eta=\frac{\rho}{\rho_0}-1 \tag{28}$$

$$p_H(\rho)=\begin{cases}a_0\eta+b_0\eta^2+c_0\eta^3 & ,\eta>0\\ a_0\eta & ,\eta<0\end{cases} \tag{29}$$

$$a_0=\rho_0 c^2 \tag{30}$$

$$b_0=a_0\left[1+2(S-1)\right] \tag{31}$$

$$c_0=a_0\left[2(S-1)+3(S-1)^2\right] \tag{32}$$

CONTACT MODEL

The contact model used here is that derived by [8] for SPH low velocity impact problem. They derived the weak form of the contact problem using the virtual work principal and solved it by penalty method that involves both a penetration and a penetration rate.

The SPH form of the variational equation for the contact problem is given by [8],

$$\sum_i^N\left[\dot{v}_i^\alpha\sum_{j\in M_i}\rho_i W_{ij}V_jV_i+\sum_{j\in M_i}\sigma_j^{\alpha\beta}\nabla W_{ij}V_jV_i+\sum_{j\in M_i}\left(\lambda_1\dot{p}_n+\lambda_2 p_n\right)\mathbf{n}_iW_{ij}A_iV_j\right]\delta v_i^\alpha=0 \tag{33}$$

where M_i represents those particles within a distance 2h of a particle i, M_i^B those boundary particles within a distance 2h of particle i, and A_i is the contact area. p_n is the penetration and $\dot{p}_n$ is the penetration rate. λ_1 denotes the penalty parameter for the penetration rate and λ_2 is the penalty parameter for the penetration. The inside of the bracket must be zero for allowable virtual velocity δv_i^α. Therefore the momentum equation for two bodies in contact can be written as [8],

$$m_i\dot{\mathbf{v}}_i=-\mathbf{F}_i^{\text{int}}-\mathbf{F}_i^{con} \tag{34}$$

where

$$m_i=\sum_{j\in M_i}\rho_iW_{ij}V_jV_i$$
$$\mathbf{F}_i^{\text{int}}=\sum_{j\in M_i}\sigma_j^{\alpha\beta}\nabla W_{ij}V_jV_i \tag{35}$$
$$\mathbf{F}_i^{con}=\sum_{j\in M_i}\left(\lambda_1\dot{p}_n+\lambda_2p_n\right)\mathbf{n}_iW_{ij}A_iV_j$$

CONTACT FORCE

The [8] Uses the one dimensional elastic wave theory to obtain the expression for contact force. Their equation for penalty parameters are,

$$\lambda_1\dot{p}+\lambda_2p=\left(\frac{\rho_jC_j}{\rho_jC_j+\rho_iC_i}\rho_iC_i\right)\dot{p}+\left(\frac{E_iE_j}{E_i+E_j}\frac{1}{d_o}\right)p \tag{36}$$

where ρ_i and ρ_j are densities of SPH particle i and j in contact, C_i and C_j are their respective sound speeds, E_i and E_j are their respective elastic moduli and d_o is the diameter of the SPH particles.

CONTACT DETECTION

To find the boundary particles a color parameter, ψ_i is introduced for each SPH particles. A particle will be designated as a boundary particle if the summation of ψ_i is less than 0.85~0.90 of the original index value [7, 8].

$$\psi_i\neq\sum_{j\in N(i)}\psi_jm_jW_{ij}/\rho_j \tag{37}$$

The boundary normal vector is obtained from the gradient of the color parameter.

$$n_i=\pm\left(\nabla\psi/\left|\nabla\psi\right|\right)_i \tag{38}$$

where

$$(\nabla\psi)_i=-\sum_{j\in M_i}\psi_jm_j\nabla W_{ij}/\rho_{ij} \tag{39}$$

The following criterion is used to detect the contact [8].

$$\left|\mathbf{r}_{ij}\right|\le\sqrt{Max\left(\frac{d_i}{2},\frac{d_j}{2}\right)^2+\left(\frac{d_i+d_j}{2}\right)^2} \tag{40}$$

where $\left|\mathbf{r}_{ij}\right|$ is the center to center distance between two SPH particles and d_i and d_j are their respective diameters.

To obtain the penetration and penetration rate for two SPH particles in contact by taking into account the curvature of the surface [8] proposes a way to find the average normal vector for the two surfaces in contact. If θ is the angle formed with the normal vectors of particle i and j then the average normal vector for the two particles in contact is given by,

$$\left|\theta\right|=\angle\left(-\mathbf{n}_i,\mathbf{n}_j\right)$$

$$\begin{aligned}&(\mathbf{i})\ \left|\theta\right|<\theta_c\quad \text{if Max}\left(\mathbf{r}_{ij}\cdot\mathbf{n}_i,-\mathbf{r}_{ij}\cdot\mathbf{n}_j,\frac{\mathbf{n}_i-\mathbf{n}_j}{\left|\mathbf{n}_i-\mathbf{n}_j\right|}\right)\\ &\qquad\qquad\text{then }\mathbf{n}_{\text{av}}=\left(\mathbf{n}_i,-\mathbf{n}_j,\frac{\mathbf{n}_i-\mathbf{n}_j}{\left|\mathbf{n}_i-\mathbf{n}_j\right|}\right)\\ &(\mathbf{ii})\ \left|\theta\right|\ge\theta_c\quad \mathbf{n}_{\text{av}}=\left(\frac{\mathbf{n}_i-\mathbf{n}_j}{\left|\mathbf{n}_i-\mathbf{n}_j\right|}\right)\\ &\qquad\qquad\text{then }\mathbf{n}_{\text{av}}=\left(\mathbf{n}_i,-\mathbf{n}_j,\frac{\mathbf{n}_i-\mathbf{n}_j}{\left|\mathbf{n}_i-\mathbf{n}_j\right|}\right)\end{aligned} \tag{41}$$

$$\theta_c\approx70^o$$

where θ_c is the critical angle. The penetration and penetration rate for contact between particles is given by

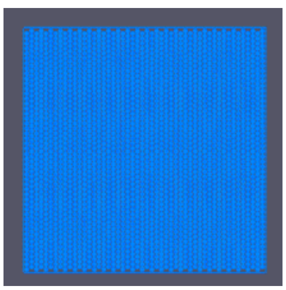
$$\begin{aligned}p_{n_{av}}&=\left(R_i+R_j-\left|\mathbf{r}_{ij}\cdot\mathbf{n}_{\text{av}}\right|\right)\\ \dot{p}_{n_{av}}&=\left(\mathbf{v}_i-\mathbf{v}_j\right)\cdot\mathbf{n}_{\text{av}}\end{aligned} \tag{42}$$

The force of contact is then obtained by,

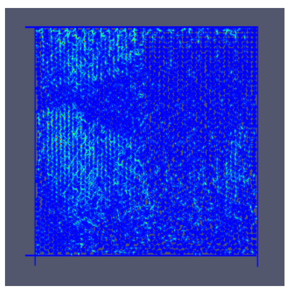
$$\mathbf{F}_i^{con} = \sum_{j\in M_i} \left(\lambda_1 \dot{p}_{n_{av}} + \lambda_2 p_{n_{av}}\right)\mathbf{n}_i W_{ij} A_i V_j \tag{43}$$

PHYSICAL MODEL

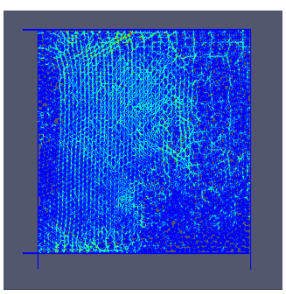
The simulation domain includes a square container with 2,400 mono-sized disks within the box. The box size, disk size and their material properties are given in Table 1 and Table 2. The disk are placed such that there is no initial contact with the wall of the container or among the disks. The initial packing fraction for the system is 0.8. Gravity is neglected in this study. Two sides of the container are kept stationary while the two other sides are displaced with a linear velocity. The sidewalls move backward and forward with an amplitude of 20 mm in each cycle. However each cycle consists of several steps where the walls are kept stationary to relax the system. The time step used for the simulation is 0.1 microsecond. The moving walls perform a series of 10 cycles where the system is quasi-statically compressed or expanded.



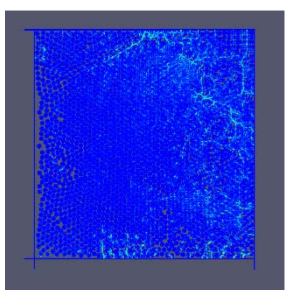
(a) Time = 0s



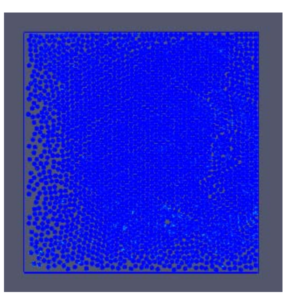
(b) Time = 7.5×10^{-4}s



(c) Time = 12.5×10^{-4}s



(d) Time = 17.5×10^{-4}s



(e) time = 25×10^{-4}s

Figure 1 One full cycle of compression and expansion showing also the force chain networks

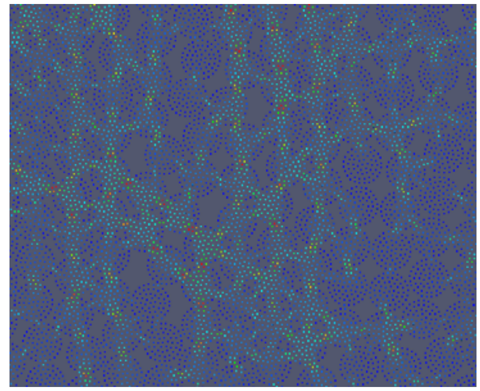


Figure 2 A closer look of the disks and force chains

Table 2 Simulation Parameters

Parameters	Values
Box size, m×m	0.2942×0.2942
Time step	1×10^{-7} s
Granular particle numbers	2400
Granular Particle radius	3.026×10^{-3} m
Number of SPH particles per disk	61
SPH Particle radius	3.4868×10^{-4} m

RESULTS AND DISCUSSION

COORDINATION NUMBER AND PACKING FRACTION

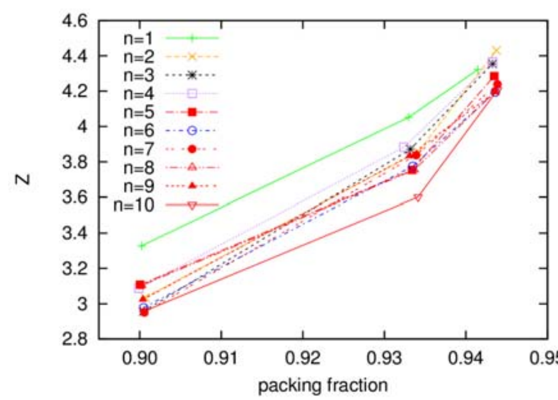


Figure 3 Average coordination number vs packing fraction

For a granular system to reach mechanical equilibrium or "jammed" the system needs to satisfy a minimum number of contacts that is theoretically related to the degrees of freedom of the system. The isostatic conjecture [14] for frictionless system of N particles in dimension d states that for mechanical equilibrium there must be at least z = 2d contacts on average per particle (since there are NZ/2 independent forces and dN force balance constraints). For the simulation performed, Figure 3 shows the average coordination number, Z for each cycle against packing fraction, ϕ . As expected Z varies around 4 as the system reaches jamming state for each cycle. Figure 3 also shows that the average number of contacts increases rapidly with packing fraction after or near isostatic jamming point. This can be compared with the pressure versus packing fraction data of Figure 4. The number of contacts increases as the disks are compressed against the walls and so is the pressure. However it can be seen that the average coordination number for the subsequent cycles are smaller which is also reflected in pressure-packing fraction diagram of Figure 4.

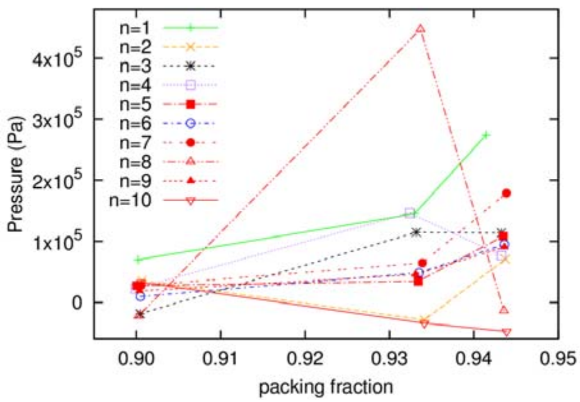


Figure 4 Global pressure vs packing fraction

GLOBAL PRESSURE RESPONSE

The global pressure on the granular system is computed by first computing the Cauchy stress tensor for the granular system. The Cauchy stress is given by [15]

$$\sigma_{ij} = \frac{1}{2A}\sum\left(F_i x_j + F_j x_i\right) \tag{44}$$

where A is the area of the confining container. F_i, F_j are the components of the concentrated force F on the boundary that is applied on the disks at points $\left(x_j, x_j\right)$. The summation is taken over all such forces. Pressure, P is then the trace of the stress tensor.

Global pressure response of the system shown in Figure 4 indicates that pressure increases gradually with packing fraction. However this increment also depends on the material properties of the particle. Figure 4 also shows support for a trend that says that pressure of the system for consecutive cycles for same packing fraction tends to be lower. Although the system is jammed during part of each cycle, the stress relaxes to somewhat lower value. This may be because the cyclic compression of the disks allows the entire structure to slowly rearrange themselves and attain a state of less global pressure for same packing fraction.

DISTRIBUTION OF CONTACT FORCES

The distribution of contact forces for different compression cycles is shown in Figure 5. Unlike DEM where the force acts at a single point, in SPH the contact force for a single granular particle is found by integrating forces on the individual SPH particles over the area of contact. Former studies [16] of granular materials have pointed out that the probability distribution of contact forces decreases exponentially with increase of contact forces. The data shown in Figure 5 also show similar trend. In fact the force distribution except for the first cycle is very similar for rest of the cycles.

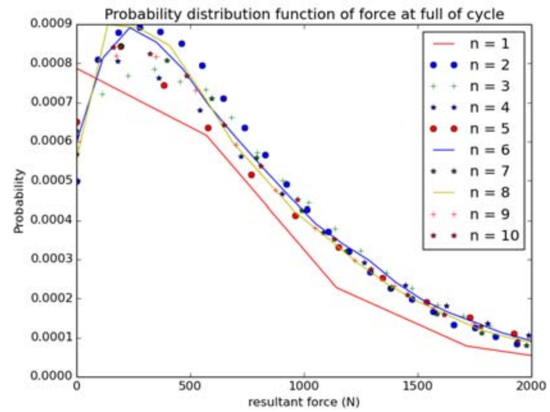


Figure 5 PDF vs resultant contact force

CONCLUSION

In this paper Smooth Particle Hydrodynamics (SPH) method is used to simulate the cyclic packing of deformable two dimensional disks and study their packing behavior. The results obtained show that the average coordination number varies with packing fraction during jamming by conforming to the isostatic conjecture. Stress relaxation is seen to occur after several compression cycles which is marked by a decrease in coordination number and global pressure. Force distribution shows similar exponential behavior as the average force on the system is increased. SPH method proves to be an important method for simulating granular materials and study their jamming behavior.

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